ATOMIC ENERGY CENTRAL SCHOOL- KAKRAPAR

CLASS-10

CHAPTER-10, CIRCLE

MODULE-2

Activity : On a paper, draw a circle and a secant PQ of the circle. Draw various lines parallel to the secant on both sides of it. You will find that after some steps, the length of the chord cut by the lines will gradually decrease, i.e., the two points of intersection of the line and the circle are coming closer and closer [see Fig. 10.3(ii)]. In one case, it becomes zero on one side of the secant and in another case, it becomes zero on the other side of the secant. See the positions P′Q′ and P′′Q′′ of the secant in Fig. 10.3 (ii). These are the tangents to the circle parallel to the given secant PQ. This also helps you to see that there cannot be more than two tangents parallel to a given secant.

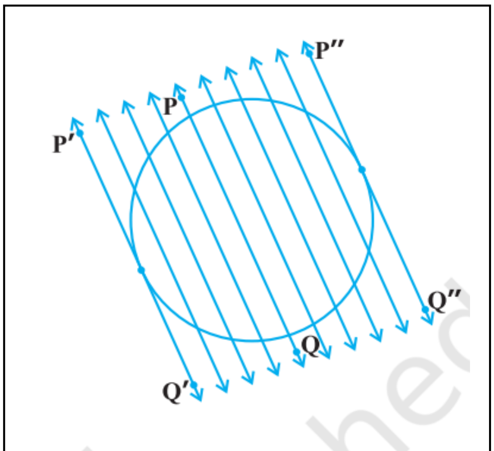

Fig. 10.3 (ii)

This activity also establishes, what you must have observed, while doing
Activity 1, namely, a tangent is the secant when both of the end points of the corresponding chord coincide.

Theorem 10.1 : *The tangent at any point of a circle is perpendicular to the radius through the point of contact.*

Proof : We are given a circle with centre O and a tangent XY to the circle at a point P. We need to prove that OP is perpendicular to XY.
Take a point Q on XY other than P and join OQ (see Fig. 10.5).
The point Q must lie outside the circle.
Therefore, OQ is longer than the radius OP of the circle.

That is, $OQ > OP$.

Since this happens for every point on the line XY except the point P, OP is the

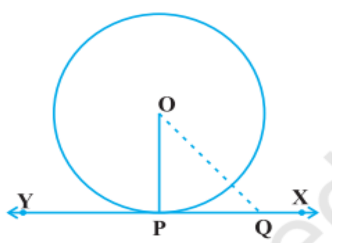

Fig. 10.5

shortest of all the distances of the point O to the points of XY. So OP is perpendicular to XY.

Theorem 10.2 : ***The lengths of tangents drawn from an external point to a circle are equal*.**

Proof : We are given a circle with centre O, a point P lying outside the circle and two tangents PQ, PR on the circle from P (see Fig. 10.7). We are required to prove that PQ = PR.
For this, we join OP, OQ and OR. Then $\angle$ OQP and $\angle$ ORP are right angles, because these are angles between the radii and tangents, and according to Theorem 10.1 they are right angles. Now in right triangles OQP and ORP, OQ = OR
OP = OP (Radii of the same circle)
Therefore, Δ OQP $\cong$ Δ ORP (RHS congruency rule)

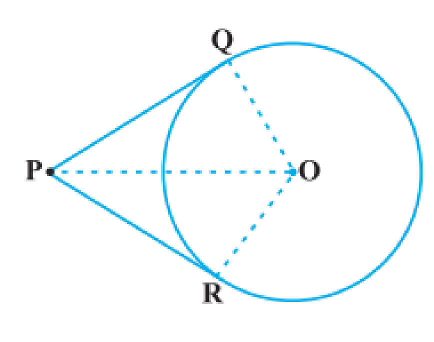

Fig. 10.7

This gives PQ = PR (CPCT)

Example-Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that $\angle$ PTQ = 2 $\angle$ OPQ.

We are given a circle with centre O,
an external point T and two tangents TP and TQ
to the circle, where P, Q are the points of contact
(see Fig. 10.9).
We need to prove that
$\angle$ PTQ = 2 $\angle$OPQ
Let PTQ = θ
Now, by Theorem ***The lengths of tangents drawn from an external point to a circle are equal***
TP =TQ so ΔTPQ is an isosceles triangle.

Therefore, $\angle TPQ = \angle TQP = \frac{1}{2}(180^0 - \theta)$

$= \frac{1}{2}(90^0 - \theta/2)$

Also we know that,The tangent at any point of a circle is perpendicular to the radius through the point of contact.

$\angle OPT = 90°$

So $\angle OPQ = \angle OPT - \angle TPQ = 90^0 - (90^0 - \theta/2)$

$= \theta/2$

That is $\angle PTQ = 2\angle OPQ$